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Baryogenesis at the Electroweak Scale

A. Kundu and S. Mallik
Saha Institute of Nuclear Physics
1/AF, Bidhannagar, Calcutta 700064, India

Abstract

The generation of the baryon asymmetry of the universe is considered in the standard model of the electroweak theory with simple extensions of the Higgs sector. The propagation of quarks of masses up to about 5 GeV are considered, taking into account their markedly different dispersion relations due to propagation through the hot electroweak plasma. It is shown that the contribution of these lighter quarks to the baryon asymmetry can be comparable to that for the t quark considered earlier.

I. INTRODUCTION

It has been an extremely interesting observation that all the conditions needed for baryogenesis [1] are already present, in principle, in the standard electroweak theory. The baryon number violation in this theory, although exceedingly suppressed at the present time [2], can be unsuppressed at high temperature [3]. C and CP violations are contained in the interaction of quarks with Higgs fields. Finally departure from thermal equilibrium, although difficult to obtain at the electroweak scale – typical weak interaction rates are extremely faster than the expansion rate of the Universe – can nevertheless exist if the electroweak phase transition is of first order. We thus have the exciting possibility of explaining one of the most fundamental problems of cosmology in terms of laboratory physics.

Several mechanisms have been proposed to obtain the observed baryon to entropy ratio in the electroweak theory [4-9]. In particular, Nelson et al [7,8] consider simple extensions of the minimal standard model (MSM) to obtain sufficient CP violation. This is provided by the complex space dependent fermionic mass function within the bubble wall, which arises quite generally in such models. The reflection and transmission coefficients are then different for particles and antiparticles, leading to a separation of some CP-odd charge. The latter is then converted to baryon asymmetry in the broken phase by the baryon number violating process in the unbroken phase. They consider only the propagation of the top quark through the medium due to its large Yukawa coupling [8].

The MSM, whose CP violation was earlier thought to be too small to generate any significant baryon asymmetry, has recently been shown by Farrar and Shaposhnikov [9] to have the potential to generate this asymmetry by the above mechanism, provided one takes the quark mixing effects at high temperature properly into account. Further they consider a direct separation of the baryon number by the bubble wall rather than of some other CP- odd charge.

Although the nonminimal models were initially studied because of the insufficiency of baryon asymmetry produced in MSM, we believe that it is worthwhile to investigate these extensions on the cosmological front, as long as one is looking for deviations from the MSM in the laboratory [10].

Here we reconsider the simply extended MSMs for lighter quarks with masses up to about 5 GeV. The dispersion relation for these quarks in the

electroweak plasma is markedly different from the free one [9,11]. It is not immediately clear how such propagations are going to affect the calculation of the baryon asymmery. Following Ref [9] we take the altered fermion propagation into account to leading order and examine the generation of this asymmetry by the so-called normal and abnormal modes.

We avoid the details of specific models by parametrising the mass function in a simple way and consider the direct separation of baryon number by the wall. We also restrict our calculation to low bubble wall velocity. The reflection coefficients of quarks from the bubble wall are obtained in an iteration series. We argue that our calculation, though oversimplified, does give the order of magnitude estimate of baryon asymmetry, subject to the uncertainties involved in the details of phase transition and other properties of the medium.

In sec.II we review the propagation properties of quark excitations in the electroweak plasma. We then calculate in sec.III the reflection and transmission currents giving rise to baryon asymmerty. These expressions are then evaluated in sec.IV. Our concluding remarks are contained in sec.V. In the appendix we solve the quark equation of motion within the wall in an iteration series to fourth order.

II.QUARK PROPAGATION IN HOT PLASMA

Here we review the propagation properties of light quark excitations in the electroweak plasma at high temperature. The most important effect of the medium on the quark is that it acquires a chirally invariant effective mass with an altered dispersion relation. Neglecting smaller contributions due to the weak gauge boson and Higgs scalar exchanges compared to that due to gluon exchange, the one loop self-energy leads to the same effective mass E_0, for both left (L)- and right (R) -handed quarks,

$$E_0 = (2\pi\alpha_s/3)^{1/2}T \approx 0.5T \tag{1}$$

with $\alpha_S = .12$ at the Z boson mass. For excitations close to E_0, the effective Lagrangian incorporating the altered dispersion relation is [9]

$$\mathcal{L} = iR^\dagger(\partial_0 + \frac{1}{3}\sigma.\nabla + iE_0)R + iL^\dagger(\partial_0 - \frac{1}{3}\sigma.\nabla + iE_0)L + mL^\dagger R + m^\star R^\dagger L \tag{2}$$

where we have also included the mass acquired through Higgs mechanism.

The Lagrangian (2) gives the equation of motion for the L and R components. In the following we consider the one-dimensional problem where quarks propagate along the z-axis, normal to the bubble wall. Writing

$$L = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, R = \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix},$$

the equations split into two independent sets. Defining

$$\Phi = \begin{pmatrix} \psi_1 \\ \psi_3 \end{pmatrix}, \Phi' = \begin{pmatrix} \psi_4 \\ \psi_2 \end{pmatrix},$$

and considering solutions with positive energy E, they are

$$\frac{d}{dz}\Phi = iQ(z)\Phi, \tag{3}$$

where

$$Q(z) = 3 \begin{pmatrix} E - E_0 & m^\star \\ -m & -(E - E_0) \end{pmatrix}, \tag{4}$$

and a similar one for Φ' with m replaced by its complex conjugate.These equations refer to the fluid rest frame. Although we work in the wall rest frame, it suffices to evaluate the reflection coefficients in the fluid rest frame, because of our restriction to linear terms in the bubble wall velocity in calculating the baryon asymmetry.

The planar bubble wall has a finite thickness, extending from $z = 0$ to $z = z_0$. It separates the broken phase ($z > z_0$) from the unbroken phase ($z < 0$). The Higgs induced mass m rises from zero in the unbroken phase through the bubble wall to the (almost) zero temperature mass m_0 in the broken phase.

Requiring plane wave solutions in the broken phase, eqn.(3) yields the altered dispersion relation mentioned above [12],

$$E_\pm = E_0 \pm \sqrt{(p/3)^2 + m_0^2} \tag{5}$$

In contrast to the free particle dispersion relation, where only one branch belongs to positive energy, the presence of $E_0 (> m_0)$ in (5) now makes both branches accessible to a particle with positive energy. The two branches are

called normal (+) and abnormal (-) modes of propagation. Unlike the energy E, the value of the variable p, however, does not give the true momentum $\bar{p}_\pm$ of the excitation, the latter being given by

$$\bar{p}_\pm = \pm \frac{p}{3|p|} E_\pm \tag{6}$$

Also the group velocities for the two branches are given by

$$v_\pm = \frac{d}{dp} E_\pm = \pm \frac{1}{3} \frac{p}{\sqrt{(p/3)^2 + m_0^2}}. \tag{7}$$

In the unbroken phase, where the dispersion relation becomes

$$E_\pm = E_0 \pm k/3, \tag{8}$$

each of the components ψ_i satisfies an uncoupled equation. $\psi_{1,2}$, belonging to chirality $+1$, has $k = \pm 3(E - E_0)$ respectively, while $\psi_{3,4}$, belonging to chirality -1, has $k = \mp 3(E - E_0)$ respectively. It might be thought that a particular component ψ_i would describe a left- or right-moving particle, depending on whether the energy belongs to the normal ($E > E_0$) or the abnormal ($E < E_0$) branch respectively. However, this is not true, as the direction of the true momentum (and also of the group velocity) is opposite to that of k in the abnormal mode.

Over the domain wall, m is space dependent. We may write the solution for $\Phi(z)$ as

$$\Phi(z) = \Omega(z)\Phi(0), 0 \leq z \leq z_0, \tag{9}$$

in terms of the 2×2 unimodular matrix $\Omega(z)$. At $z = z_0$ its elements will be denoted by

$$\Omega(z_0) = \begin{pmatrix} \alpha & \beta \\ \beta^\star & \alpha^\star \end{pmatrix}. \tag{10}$$

An iterative solution is obtained in the Appendix.

We note here the Lorentz invariant expression for the density of fermionic excitations,

$$n = (\exp \beta p \cdot v + 1)^{-1}.$$

Here β is the inverse temperature of the fluid in the frame where it is at rest, p^μ is the energy momentum 4-vector of the excitation and v^μ , the 4-velocity

of the medium. In the wall rest frame, $p^\mu = (E, \bar{p})$, $v^\mu = \gamma(1, v)$ where $\gamma = 1/\sqrt{1 - v^2}$ and $\bar{p}$ is the true momentum given by (6). For p along the positive z-direction, we thus have in this frame, $p \cdot v = E_\pm(1 \mp v/3)$, up to linear term in v. In the following we need the densities of particles moving towards the wall. In the unbroken phase these are given by

$$n^u_\pm = \frac{1}{e^{\beta E_\pm(1-v/3)} + 1} \tag{11}$$

for the $(\pm)$ modes respectively. In the broken phase the corresponding quantities $n^b_\pm$ are given by the same expressions with the reversal of sign before v.

III. REFLECTION AND TRANSMISSION CURRENTS

Since baryon non-conservation through the sphaleron processes involves the left-handed fermions and antifermions, we are interested in calculating the left-handed baryonic currents only.

Consider first the propagation of quark excitation by the normal mode. We send a right-handed fermion towards the domain wall from the unbroken phase. Noting the revarsal of chirality at the wall, the incident wave (of unit current) and the reflected wave of amplitude r , say, is given by

$$\Phi(z) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ikz} + \begin{pmatrix} 0 \\ r \end{pmatrix} e^{-ikz}, z \leq 0. \tag{12}$$

On the right (broken phase), we have only the transmitted wave of amplitude t, say. Solving eqn.(3) for Φ we get

$$\Phi(z) = t \begin{pmatrix} \cosh\theta \\ -\sinh\theta \end{pmatrix} e^{ip(z-z_0)}, z \geq z_0. \tag{13}$$

Here p satisfies eqn.(5) with the plus sign and

$$\cosh\theta = \sqrt{\frac{E - E_0 + p/3}{2p/3}}.$$

To find the unknown ampltudes we use the boundary conditions given by eqn.(9) for $z = z_0$,

$$t \begin{pmatrix} \cosh\theta \\ -\sinh\theta \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \beta^\star & \alpha^\star \end{pmatrix} \begin{pmatrix} 1 \\ r \end{pmatrix}. \tag{14}$$

The reflection coefficient is

$$R_+ = |r|^2 = 1 - \frac{1}{|\alpha^\star \cosh\theta + \beta \sinh\theta|^2}. \tag{15}$$

The incident current, same for particles and antiparticles, is $\frac{1}{3} n_+^u$, where n_+^u is given by (11). Then the net contribution to the reflected left-handed baryonic current is

$$\int_{3m_0}^{\infty} \frac{dk}{2\pi} \frac{1}{3} n_+^u (R_+ - \bar{R}_+) \tag{16}$$

Here and in the following a bar on a reflection or transmission coefficient denotes the corresponding quantity for the antiparticle. It is obtained by solving the same eqn.(3) with m replaced by $m^\star$.

Next we calculate the transmitted baryonic current in the unbroken phase due to incidence on the wall from the broken phase. On the left there is simply a transmitted wave of amplitude $\tilde{t}$, say,

$$\Phi(z) = \begin{pmatrix} 0 \\ \tilde{t} \end{pmatrix} e^{-ikz}, z < 0 \tag{17}$$

On the right we have both the incident wave (of unit current) and the reflected wave of amplitude $\tilde{r}$, say. Solving eqn.(3) separately for the two cases, we get

$$\Phi(z) = \begin{pmatrix} \sinh\theta \\ -\cosh\theta \end{pmatrix} e^{-ip(z-z_0)} + \tilde{r} \begin{pmatrix} \cosh\theta \\ -\sinh\theta \end{pmatrix} e^{ip(z-z_0)}, z \geq z_0 \tag{18}$$

Again using the boundary condition (9) at $z = z_0$, we have

$$\begin{pmatrix} \sinh\theta + \tilde{r}\cosh\theta \\ -(\cosh\theta + \tilde{r}\sinh\theta) \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \beta^\star & \alpha^\star \end{pmatrix} \begin{pmatrix} 0 \\ \tilde{t} \end{pmatrix} \tag{19}$$

The transmission coefficient is then

$$T_+ = |\tilde{t}|^2 = 1 - R_+ \tag{20}$$

There arises a transmitted left-handed baryonic current in the unbroken phase given by

$$\int_0^{\infty} \frac{dp}{2\pi} \frac{1}{3} \frac{p}{k} n_+^b (T_+ - \bar{T}_+) \tag{21}$$

We add (15) and (20) to get the total normal mode baryonic current in the unbroken phase due to reflection and transmission as

$$J_+ = \int_{3m_0}^{\infty} \frac{dk}{2\pi}\frac{1}{3}(n^b_+ - n^u_+)(T_+ - \bar{T}_+) \tag{22}$$

A similar contribution to the baryonic current results from propagation by the abnormal mode in the energy region $E < E_0 - m_0$. It is given by

$$J_- = \int_{3m_0}^{3E_0} \frac{dk}{2\pi}\frac{1}{3}(n^b_- - n^u_-)(T_- - \bar{T}_-) \tag{23}$$

where

$$T_- = \frac{1}{|\alpha^* \cosh\theta' - \beta \sinh\theta'|^2} \tag{24}$$

with

$$\cosh\theta' = \sqrt{\frac{E_0 - E + p/3}{2p/3}}.$$

The total CP-violating left-handed baryonic current in the unbroken phase generated by reflection from and transmission through the bubble wall of the fermionic excitations by the normal and the abnormal modes is

$$J^L_{CP} = J_+ + J_- \tag{25}$$

The final step is to obtain the baryonic density n_B in the broken phase from the steady state solution to the rate equations in the two phases. Nelson et al find numerical solution to the Boltzmann equations. We shall follow Farrar and Shaposhnikov, who solve the diffusion equations for small bubble wall velocity to get

$$n_B = J^L_{CP} f \tag{26}$$

where f is a given function of the diffusion coefficients for quarks and leptons, the wall velocity and the sphaleron induced baryon number violation rate. Their estimate for f is $10^{-3} \le f \le 1$ in MSM, which should also be valid for its simple extensions.

IV. ESTIMATION

In the standard model with a single Higgs doublet, the expectation value of the Higgs field is real everywhere during phase transition. Adding extra multiplets will allow in general some of their components to acquire space-dependent complex values within the bubble wall. This in turn leads to a complex mass function for the quark having Yukawa coupling to those multiplets. It is, in principle, derivable from the model considered but, in practice, will depend on the (unknown) Higgs couplings. Here we assume the simplest form for the mass function,

$$m(z) = \frac{m_0}{z_0} z + i\frac{\delta}{z_0^2} z(z_0 - z) \tag{27}$$

within the bubble wall. The parameter δ is related to the magnitude of CP violation in the model.

With the parametrization (27), we solve eqn.(3) by iteration. This solution becomes a power series in three dimensionless quantities, viz, the energy variable $y = 3(E - E_0)z_0$ and the constants $c = 3m_0 z_0$ and $d = 3\delta z_0$. In the Appendix we have obtained this series up to fourth order. Inspection of the coefficients suggests that for y, c and d less than unity, it should represent the solution well.

The condition $y < 1$ restricts k to $k < 1/z_0$. Now the reflection coefficients are known to be asymptotically $\sim exp(-2\pi|k|z_0)$ which appears to set in already for $k > 1/z_0$. Our approximation then consists of truncating the upper limit of k integration at $k = 1/z_0$, within which we use our solution to compute the reflection coefficients.

As long as the wall thickness $z_0 \leq (15GeV)^{-1}$ [13,14], the condition $c < 1$ would allow quarks with masses up to that of the b quark. Note that even if this condition had allowed the t-quark mass, our result would not apply to this quark propagation, since the dispersion relation on which we base our work, would not be valid. Instead, the free particle dispersion relation would be more appropriate for the t-quark, as has been assumed in the work of Nelson et al. Finally the condition $d < 1$ or $\delta < 1/3z_0$ is a very reasonable bound for the imaginary part of the quark mass within the bubble wall.

It is now simple to evalulate the currents $J_\pm$ given by eqns. (21,22). In the following we choose $z_0 \simeq (20GeV)^{-1}$, whence the upper limit of k integration is $\simeq 20GeV$. As the temparature of the phase transition $\sim 100GeV$, we may

expand the density function in v for small v and approximate the exponentials by unity to get

$$n_{\pm}^{b} - n_{\pm}^{u} \simeq -\frac{1}{6}\beta E_{\pm} v. \tag{28}$$

The transmission coefficient may be calculated in a straightforward way using the values of α and β given in the Appendix. We get

$$T_{\pm} - \bar{T}_{\pm} = \mp 4dc \frac{\sqrt{y^2 - c^2}(1/3 - 4y^2/45 + 5c^2/42)}{\{(1 - c^2/6 - c^4/168)y + c^2y^3/45 + \sqrt{y^2 - c^2}\}^2} \tag{29}$$

Clearly for $c < 1$, the higher powers of c can be safely neglected. Thus

$$J_{\pm} = \pm \frac{\beta v d c}{27\pi z_0} \int_c^1 dy \frac{(E_0 \pm y/3z_0)\sqrt{y^2 - c^2}(1 - 4y^2/15)}{(y + \sqrt{y^2 - c^2})^2} \tag{30}$$

Observe the large cancellation in the sum of J_+ and J_- arising from quark propagation by the normal and abnormal modes respectively. The asymmetry current becomes

$$\begin{aligned} J_{CP}^{L} &= \frac{2\beta v d c}{81\pi z_0^2} \int_c^1 dy \frac{y\sqrt{y^2 - c^2}(1 - 4y^2/15)}{(y + \sqrt{y^2 - c^2})^2} \\ &\sim \frac{1}{18\pi}\beta v \delta m_0 \end{aligned} \tag{31}$$

Finally noting the one dimensional entropy density $s = 73\pi/3\beta$, the baryon to entropy ratio is given by

$$n_B/s \sim 10^{-8} v f \delta m_0 \tag{32}$$

where δ and m_0 are expressed in GeV. Within the framework of our calculation, it is the dynamics of the b quark propagation through the phase transition bubbles, which gives a sizeable contribution to n_B/s. With $v \sim 0.1$ and δ not too small, its contribution can be comparable with the observed asymmetry, $n_B/s \sim 5 \times 10^{-11}$.

V. CONCLUSION

We have investigated the generation of baryon asymmetry in the standard model of the electroweak theory with more than one Higgs multiplet. Such models generally give rise to a complex mass function for a quark within the wall of bubbles formed during the phase transition. This constitutes the CP violation needed for baryogenesis. We do not attempt to calculate such a mass function, however: We paramertise its real and imaginary parts in a simple way, making the integrals trivial to evaluate. We follow Ref [9] to consider the direct separation of baryon number by the phase boundary rather than separation of some other CP-odd charge to be converted into baryon asymmetry by a separate process, as discussed in Ref [8].

The inclusion of the temperature dependent effective mass gives rise to two modes of quark propagation in the plasma. Our calculation shows that both modes must be considered. In fact, the net baryon asymmetry current results after large cancellation between the baryonic currents carried separately by the two modes.

The calculation presented here is a simple analytic one giving the order of magnitude estimate of the baryon asymmetry. Our formula (32) neatly isolates the model dependent parameters involved in the description of the electroweak medium and the first order phase transition. These are the wall thickness z_0 [15], the velocity v of the medium, δ giving the magnitude of the imaginary part and f related to the plasma diffusion and sphaleron transition rate.

The calculation is limited to small bubble wall velocity in the rest frame of the plasma, as we keep only the terms linear in v. Also it concerns lighter quarks for which the dispersion relations deviate appreciably from those of free propagation. For the t quark the free particle dispersion relation is accurate enough and its propagation can be discussed following the treatment of Nelson et al. Our calculation indicates that the contribution of lighter quarks, like the b quark, to the baryon asymmetry can also be substantial.

After completing the work, we learnt of several works [16,17] taking into account the effect of damping in the quark propagation. These authors have shown that it reduces the reflection coefficient to negligible values making the present mechanism totally ineffective to reproduce the observed baryon asymmetry in the minimal standard model of the electroweak theory. However, this conclusion is not agreed by others [18], who claim that the damping

rate is irrelevant to the problem.

Even if we take the damping into account, it is easy to see that it cannot drastically affect our result obtained in nonminimal models. In the minimal model, the baryon asymmetry formula involves high powers of the mass matrix. Damping effectively multiplies each such matrix by a suppression factor, making the asymmetry negligible. However, in our formula the mass function appears only quadratically. Furthermore, the range of values allowed for the wall thickness z_0 is expected to be large in nonminimal models. For $z_0 \leq (20GeV)^{-1}$, a simple estimate indicates that the total suppression factor cannot be smaller than 10^{-3}. Inclusion of this factor still keeps our result (32) for n_B/s near the observed value, given the large uncertainty in the value of f mentioned at the end of sec.III.

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APPENDIX

We solve eqn. (3) by iterating the corresponding integral equation,

$$\Phi(z) = \Phi(0) + i\int_0^z dz_1 Q(z_1)\Phi(z_1) \tag{33}$$

For the matrix $\Omega(z)$ in eqn. (9) the solution reads

$$\Omega(z) = 1 + i\int_0^z dz_1 Q(z_1) + i^2\int_0^z dz_1\int_0^{z_1} dz_2 Q(z_1)Q(z_2) + \cdots \tag{34}$$

It is convenient to express $Q(z)$ in terms of Pauli matrices $\sigma^m (m = 1, 2, 3)$,

$$Q(z) = f_m(z)\sigma^m$$

with

$$f_1 = -i\mathrm{Im}\ m, f_2 = i\mathrm{Re}\ m, f_3 = E - E_0$$

With repeated use of

$$\sigma^m\sigma^n = \delta^{mn} + i\epsilon^{mnp}\sigma^p$$

we can reduce the products of σ^m to a linear combination of σ^m and the unit matrix. Writing

$$\Omega(z) = \omega_0(z)1 + \omega_m(z)\sigma^m$$

we obtain the iterative solution for the matrix elements up to fourth order,

$$
\begin{aligned}
\omega_0(z) &= 1-\int_0^z dz_1\int_0^{z_1} dz_2\mathbf{f}(z_1)\cdot\mathbf{f}(z_2)\\
&+ \int_0^z dz_1\int_0^{z_1} dz_2\int_0^{z_2} dz_3\epsilon^{mnp}f_m(z_1)f_n(z_2)f_p(z_3)\\
&+ \int_0^z dz_1\int_0^{z_1} dz_2\int_0^{z_2} dz_3\int_0^{z_3} dz_4\{\mathbf{f}(z_1)\cdot\mathbf{f}(z_2)\mathbf{f}(z_3)\cdot\mathbf{f}(z_4)\\
&- \mathbf{f}(z_1)\cdot\mathbf{f}(z_3)\mathbf{f}(z_2)\cdot\mathbf{f}(z_4)+\mathbf{f}(z_1)\cdot\mathbf{f}(z_4)\mathbf{f}(z_2)\cdot\mathbf{f}(z_3)\} \qquad (35)
\end{aligned}
$$

$$
\begin{aligned}
\omega_m(z) &= i\int_0^z dz_1f_m(z_1)-i\int_0^z dz_1\int_0^{z_1} dz_2\epsilon^{mkl}f_k(z_1)f_l(z_2)\\
&- i\int_0^z dz_1\int_0^{z_1} dz_2\int_0^{z_2} dz_3\{f_m(z_1)\mathbf{f}(z_2)\cdot\mathbf{f}(z_3)-f_m(z_2)\mathbf{f}(z_1)\cdot\mathbf{f}(z_3)\\
&+ f_m(z_3)\mathbf{f}(z_1)\cdot\mathbf{f}(z_2)\}+i\int_0^z dz_1\int_0^{z_1} dz_2\int_0^{z_2} dz_3\int_0^{z_3} dz_4[\epsilon^{mpq}\\
&\quad \{f_p(z_1)f_q(z_4)\mathbf{f}(z_2)\cdot\mathbf{f}(z_3)-f_p(z_2)f_q(z_4)\mathbf{f}(z_1)\cdot\mathbf{f}(z_3)\\
&+ f_p(z_3)f_q(z_4)\mathbf{f}(z_1)\cdot\mathbf{f}(z_2)\}+\epsilon^{abc}f_a(z_1)f_b(z_2)f_c(z_3)f_m(z_4)], \qquad (36)
\end{aligned}
$$

Here $\mathbf{f}(z_1)\cdot\mathbf{f}(z_2)=f_m(z_1)f_m(z_2)$, for example. With the parametrization (26) it is easy to evaluate the integrals. We actually need $\omega_{0,m}(z_0)$ which we simply denote by $\omega_{0,m}$.Using the variables y, c and d introduced in the text and rejecting powers of δ higher than the first, eqns.(32,33) give the following expressions ($\omega=i\hat{\omega}_3$)

$$
\begin{aligned}
\omega_0 &= 1-\frac{1}{2}y^2+\frac{c^2}{8}+\frac{1}{24}y^4-\frac{17c^2}{720}y^2+\cdots\\
&\quad +d(-\frac{c}{180}y+\cdots) \qquad (37)\\
\omega_1 &= \frac{c}{6}y-\frac{c}{60}y^3+\frac{3c^3}{560}y+\cdots+\\
&\quad +d(\frac{1}{6}-\frac{1}{60}y^2+\frac{13}{1680}c^2+\cdots) \qquad (38)\\
\omega_2 &= -\frac{c}{2}+\frac{c}{12}y^2-\frac{c^3}{48}+\cdots\\
&\quad d(\frac{c^2}{2520}y+\cdots) \qquad (39)
\end{aligned}
$$

$$
\begin{aligned}
\hat{\omega}_3 &= y - \frac{1}{6}y^3 + \frac{7c^2}{120}y + \cdots \\
&\quad + d(\frac{c}{30} - \frac{c}{315}y^2 + \frac{13c^3}{15120} + \cdots)
\end{aligned}
\tag{40}
$$

The matrix elements of $\Omega(z_0)$ in eqn.(10) are given by

$$
\alpha = \omega_0 + i\hat{\omega}_3, \beta = \omega_1 - i\omega_2
$$

References

[1] A.D.Shakharov, JETP Lett. **6**, 24 (1967)

[2] G.'t Hooft, Phys. Rev. Lett.,**37**, 8 (1976); Phys. Rev. **D14**, 3432 (1976)

[3] V.A.Kuzmin,V.A.Rubakov and M.E.Shaposhnikov, Phys. Lett. **155B**, 36 (1985). See also
V. Soni, Phys. Lett. **B93** ,101(1980)
N.S. Manton, Phys. Rev. **D28**, 2019 (1983)
F. R. Klinkhammer and N. S. Manton, Phys. Rev. **D30**, 2212 (1984) P. Arnold and L. McLerran, Phys. Rev. **D36**, 581 (1987)

[4] M.E. Shaposhnikov, Nucl. Phys. **B299**, 797 (1988)

[5] N. Turok, L. McLerran, M.E. Shaposhnikov and M.B. voloshin, Phys. Lett. **256B**, 451(1991)

[6] R. Singleton, M. Dine, P. Huet and L. Susskind, Phys. Lett. **257B**, 351 (1991)

[7] A.G. Cohen, D.B. Kaplan and A.E. Nelson, Phys. Lett. **B245**, 561 (1990) , Nucl. Phys. **B349**, 727 (1991), Phys. Lett. **B294**, 57 (1992)

[8] A.E. Nelson, D.B. Kaplan and A.G. Cohen, Nucl. Phys. **B373**, 453 (1992)

[9] M.E. Shaposhnikov, Phys. Lett. **B277**, 324 (1992), ibid. **B282**, 483(E), (1992)
G.R. Farrar and M.E. Shaposhnikov, Phys. Rev. Lett. **70**, 2833 (1993); Phys Rev. **D50**, 774 (1994)

[10] J.F. Gunion, H.E. Haber, G.L. Kane and S. Dawson, The Higgs Hunter's Guide (Addison-Wesley, Reading MA, 1990)

[11] H.A. Weldon, Phys. Rev. **D26**, 2789 (1982)

[12] In Ref.[9] nonleading contributions to the self energy are included making the effective mass different for the left- and right- handed fermions. This leads to four branches in the dispersion relation instead of two considered here.

[13] M. Dine, R. Leigh, P. Huet, A. Linde and D. Linde Phys. Rev. **D46**, 550 (1992)

[14] N. Banerjee, S. Mallik and D. Rai Chaudhuri, Phys. Rev. **D47**, 4277 (1993)

[15] The dependence on z_0 in (32) is implicit through the conditions that y, c and d are all less than unity.

[16] M. B. Gavela, P. Hernandez, J. Orloff and O. Pene, Mod. Phys. Lett. **9A**, 795 (1994)
M. B. Gavela, P. Hernandez, J. Orloff, O. Pene and C. Quimbay, preprint CERN-TH. 7263/94.

[17] P. Huet nad E. Sather, preprint SLAC-PUB-6479

[18] G. R. Farrar and M. E. Shaposhnikov, preprint RU-94-90